Adding Oxygen to the Model of Accelerated Eutrophication in a Lake with Nitrogen and Phosphorous Limitations

Background

Accelerated eutrophication of lakes is usually associated with excessive inflows of nutrients such as phosphates, nitrates and ammonium. The flow of nutrients into the lake can allow for rapid increase in biomass (i.e., algae). The algae are relatively short-lived, and the lake soon exhibits high concentration of dead organic matter (i.e., detritus). The detritus sinks to the hypolimnion, the lower layer of the lake. As the detritus gradually decays, the nitrogen and phosphorous are released back into the nutrient pool. This nutrient cycling was the focus of the previous BWeb exercises.

Now we turn our attention to the oxygen that is consumed by the decay of detritus. If the decay occurs when the lake is stratified, the lost oxygen cannot be replaced by mixing with the upper layer. And if the detritus load is heavy, the oxygen consumption could deprive the lower layer of the oxygen needed for fish.

The Oxygen Exercises

These exercises present a simple model of the dissolved oxygen in the lake. The oxygen model may then be connected to the previous model of nutrient cycling. Results of the combined model are presented to show the oxygen consequences of the previous simulations. The new graphs show that oxygen would fall to dangerously low levels during the months when the lake is stratified. This document concludes with exercises for you to construct and verify the oxygen results. The exercises then challenge you to include additional relationships to represent the reduction in the detritus decay when there is insufficient oxygen to support the decay.

The final set of exercises deal with nitrogen accumulation in the sediments. Some of the nitrogen is absorbed in the sediments, and the sediments act as a nitrogen sink when there is plenty of oxygen in the hypolimnion. Under hypoxic conditions, however, nitrogen in the upper layer of the sediments can return to the lake. This unexpected source of nitrogen is sometimes called internal loading (Wetzel 2001). If biomass growth is limited by nitrogen, the internal loading might trigger a vicious circle of growing nitrogen, increased biomass, increased detritus, greater detritus decay, greater oxygen consumption, less oxygen, and still further release of nitrogen from the sediments.

The goal of the exercises is to develop and test an expanded model for simulating watershed management policies. For example, the expanded model could be used to simulate buffer zones that reduce the nutrient inflow and help ensure adequate oxygen for the fish. The model could also show whether reduced nutrient inflows would allow the lake to avoid hypoxic conditions and the onset of internal loading. Alternatively, the expanded model could be used to test the effect of aeration to rebuild the oxygen levels in a lake that is suffering from hypoxic conditions.
Description of the Oxygen Model

Fig. 1 shows the model of dissolved oxygen in the lake. The variables in the top of the diagram deal with the epilimnion, the upper layer of the lake. The temperature and oxygen in the upper layer are treated as exogenous variables with nonlinear lookups to account for changes during the year. The timing of stratification is also treated exogenously. The simulations shown here assume that stratification begins at the end of June and is terminated by fall turnover at the end of September.¹

![Diagram of Oxygen Model](image)

Figure 1. New variables to simulate dissolved oxygen in the lake.

¹ The model represents a monomictic lake, a lake that stratifies once in the summer due to thermal inputs from the sun. The model simulates conditions in the epilimnion and the hypolimnion, but it ignores the metalimnion (the middle layer where temperature changes rapidly with depth). Further information on stratification and dissolved oxygen is provided by Water on the Web (2011).

The model is not suitable for the dimictic lakes found in colder regions. These lakes ice over in the winter, and there is no oxygen exchange between the lake and the atmosphere. Also, there is no wind-driven mixing during the iced-over months. These lakes have two turnovers, one in the spring and the second in the fall.
The variables in the lower part of Fig. 1 simulate the dissolved oxygen in the hypolimnion, the lower layer of the lake. A stock variable is used to accumulate the flows due to mixing, net production and consumption. The stock is measured in umol/L of O2, and the initial value is 395 umol/L. The model also reports the lower layer concentration in mg/L (milligrams per liter) since this is more familiar to some readers. With an atomic mass unit (AMU) of 32 ug/umol for O2, the initial concentration of the lower layer is $395 \text{umol/L} \times \frac{32 \text{ug}}{\text{umol}} \times \frac{1 \text{mg}}{1000 \text{ug}} = 12.6 \text{mg/L}$.

Figure 1 shows an additional variable to remind us of the threshold value for fish to survive. We set the threshold value at 5 mg/L. According to Hatch (2011), the lowest dissolved oxygen levels at which various fish species can survive in the summer range from 3.3 to 6.0 mg/L. The lake may be described as “putting the squeeze on fish” as depicted in the illustration below. The epilimnion is too warm and the hypolimnion has too little oxygen. The fish may end up in the upper layer where they face warmer temperatures and greater exposure to predators. The end result can be major fish mortality, as depicted in the photo from the UNEP (2011).

![Sketch of the summer squeeze on fish](Water on the Web 2011)

![Fish deaths photo](UNEP 2011)

The mixing flux in Fig. 1 is based on the difference between oxygen in the upper and lower layers. This difference is called the Hypolimnion O2 deficit. If the deficit is positive, the mixing flux will transfer oxygen into the hypolimnion. If the deficit is negative, the mixing flux will remove oxygen from the hypolimnion. The equations are:

$$O2 \text{ mixing flux} = \text{Hypolimnion O2 Deficit} \times \text{mixing feasible?} \times \text{Mixing Rate}$$

$$\text{Mixing feasible?} = 1 - \text{stratified?}$$

$$\text{Mixing Rate} = 20$$

A mixing rate of 20/month is equivalent to around 65%/day. This rapid rate will ensure that the mixing flux acts quickly to eliminate the hypolimnion deficit after the fall turnover.

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2 The units of mg/L also happen to be identical to ppm (parts per million).

3 There are no fish in the model, so the threshold value is not used elsewhere. Rather, it is a “display variable” which will be used in graphs.
The consumption of oxygen is based on the detritus decay.\(^4\) Detritus is dead organic matter which tends to sink to the hypolimnion, and we assume that 70\% of the decay occurs in the lower layer.\(^5\) The oxygen-carbon molar ratio is set at 32/12 which is 2.67. Therefore, every umol of detritus C decay consumes 2.67 umol of oxygen. Oxygen is produced by the net growth of biomass C that occurs in the lower layer. The word *net* refers to the primary production (photosynthesis) minus the oxygen consumed by respiration. Most of the biomass growth occurs in the upper layer due to higher exposure to sunlight. We assume that only 10\% of biomass growth will occur in the lower layer.\(^6\) Each umol of net biomass C growth will produce 2.67 umol of oxygen which is added to the stock of oxygen in the lower layer.

**Exogenous Assumptions: Oxygen in the Upper Layer**

Figure 2 shows the assumed patterns of temperature and dissolved oxygen in a 24-month simulation. Figure 3 shows the nonlinear lookup for the dissolved oxygen as a function of water temperature. The first two months of each year have an average temperature of 6° C. We expect the saturation concentration to be 395 umol/L (around 12.6 mg/L) in these cold months.\(^7\) The water temperature climbs in the summer, reaching a peak of 24 °C by the end of August. The dissolved oxygen in the epilimnion will be at the lowest value of the year: 263 umol/L (around 8.4 mg/L).

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\(^4\) The detritus decay in Fig. 1 is a shadow variable, a copy of a variable defined elsewhere. The original version of the detritus decay is shown in Fig. 2 of the previous BWeb document.

\(^5\) The most easily degradable carbon is broken down while still in the upper layer. The more recalcitrant carbon ends up sinking to the bottom. Overall, the fraction of detritus that decays in the hypolimnion could range from 50\% to 90\%. This uncertain input is the subject of sensitivity testing exercises.

\(^6\) The 10\% means that 90\% of the growth occur in the upper layer (due to better access to sunlight). This model input is uncertain and could be the subject of sensitivity testing.

\(^7\) Colder water contains more dissolved oxygen because it is possible for the oxygen molecules to remain trapped in the aqueous solution when the water molecules are less energized. The relationship between temperature and oxygen saturation is nonlinear. Other factors (i.e., elevation of the lake and solutes in the lake) can alter the relationship. Further information is given by Weiss (1970) and by the Water on the Web (2011).
Illustrative Results of the Expanded Model

Fig. 4 shows the simulated results for carbon, nitrogen and phosphorous in the lake. This is the same result shown in Fig. 1 of the previous BWeb document. The new model includes orange navigation to view the O2 stocks & flows (Fig. 1) and the O2 results (Fig. 5).

![Stacked Graphs for Carbon, Nitrogen and Phosphorous (umol/L)](image)

Navigate to different parts of the model:

- Stocks and Flows of C, N & P
- Biomes Growth Rate
- Verify M.M. Curves
- Stocks & Flows for O2
- Results for O2

Fig. 4. Main view of the expanded model with new navigation in orange.

![Oxygen Storage and Oxygen Flows](image)

Figure 5. Oxygen results from the expanded model.
The left-side graph in Fig. 5 shows the oxygen stored in the epilimnion and in the hypolimnion. The binary variable “Stratified?” is depicted at the bottom of the graph to remind us that the lake becomes stratified in July, August and September of each year. The seasonal variations in the upper layer are the same as shown in Fig. 2. These are exogenously specified changes due to cold water in the winter and warm water in the summer. The dissolved oxygen in the lower layer falls below the upper layer concentrations during the summer months when the lake is stratified. The summer decline becomes more and more severe as the lake becomes more heavily laden with detritus. By the 4th year, the oxygen in the hypolimnion falls to less than 20 umol/L by the end of September.

The right-side graph in Fig. 5 shows the flows that control the oxygen stored in the hypolimnion. Oxygen consumption shows a seasonal pattern with peaks in November of each year. The peak monthly consumption reaches around 165 umol/month by the end of the simulation. The more important result in Fig. 5 is the relative value of consumption and net production during the stratification period. Comparing the red curve (consumption) and the blue curve (net production) shows that the consumption far exceeds the net production during the three months when the lower layer is unable to mix with the upper layer. The high consumption is responsible for the sharp drop in dissolved oxygen. Fig. 5 shows a spike in the mixing flux when the lake turns over at the end of September each year. This mixing is responsible for the rapid return of dissolved oxygen in the hypolimnion to the saturation value of the epilimnion.

Fig. 6 shows the hypolimnion oxygen in mg/L compared to a 5 mg/L threshold assumed for fish survival. The new graph alerts us that the fish would sense dangerous conditions at the end of the stratified interval in the first year. The fish problem is more difficult in the second year, and there is very little expectation for fish survival in the final two years. Indeed, the oxygen levels at the end of the stratification season are only 0.45 mg/L. This is far below the fish survival threshold and well below the 2 mg/L sometimes used as a cutoff for anoxic conditions.
The dramatic declines in oxygen shown in Fig. 6 reveal a serious environmental problem for the simulated lake. Something needs to be done to prevent the oxygen from falling below the 5 mg/L threshold! Perhaps the fish problems can be avoided by cutting the N-inflow to the lake. This idea is the subject of a policy exercise below.

The dramatic declines in oxygen also reveal a problem with the way detritus decay is simulated in the model. The model assumes that the decay is based on a user-specified decay rate, regardless of the amount of oxygen in the hypolimnion. However, the lack of oxygen shown in Fig. 6 would lead to a smaller chance for decomposers to find the oxygen needed to fuel the aerobic decomposition of the detritus. Adding this feedback effect is left for you as one of the exercises below.

Oxygen Exercises

1. Oxygen in the upper layer: Build the model shown in Fig. 7 based on the information shown in Figs 2 and 3. The Lookup for Temp can be specified to give the approximate values for temperatures in Fig. 2. The Lookup for O2 saturation is shown in Fig. 3. Simulate the model for 24 months with DT = 0.25 and verify that you get the results in Fig. 2.

![Fig. 7. Oxygen in the epilimnion depends on the water temperature.](image)

2. Stratification: Build the model shown in Fig. 8, and set the month of stratification to 6.0 and the month of fall turnover at 9.0. (Recall that time starts at 0.0 and the month of the year starts at 0.0 when we begin the new year. When you see month of the year = 1.0, you know that we have simulated the month of January are about to start with February. Similarly, month of the year = 6.0 means we have simulated June and are about to start with July.)

![Figure 8. Equations for stratification in July, August and September.](image)
The variables \( m_1 \) and \( m_2 \) are short-named versions of the previous variables. The short names are more convenient when we use the IF THEN ELSE function to define the binary variable for stratification:

\[
\text{Stratified?} = \text{IF THEN ELSE} (m_1 < \text{month of the year} \text{ AND} \text{ month of the year} < m_2, 1, 0)
\]

Simulate the model for 24 months with \( DT = 0.03125 \) and verify that \( \text{Stratified?} \) equals 1 during the stratification period each year.

3. Verify the Results for Dissolved Oxygen:

Build the model shown in Fig. 1. The temperature and oxygen in the upper layer are based on the 1st exercise: the stratification is based on the 2nd exercise. Remember to set the initial O2 in the Hypolimnion at 395 and assign this named variable to the initial value of the stock.\(^8\) Simulate the model for 48 months with \( DT = 0.03125 \) months and the decay rate at 0.2/month. Verify that you get the results in Figs. 5 and 6.

4. Why is \( DT \) so Small? The \( DT \) in the previous exercise is 1/32\(^{\text{nd}}\) of a month, much smaller than the 1/8\(^{\text{th}}\) used in the previous model of the lake. With the new \( DT \), we need 32 steps to simulate a single month, and the 48-month simulation would require over 1,500 steps. And if we were to simulate a more typical eutrophication process, the simulation might run for 40 years and require over 15,000 steps. Recall the advice on page 44 of the book that we should think twice about simulations that require over 1,000 steps. What aspect of the oxygen model forces us to select such a small \( DT \)? Can you think of a way to alter the model that would allow simulations with a larger value of \( DT \)? Would your alteration cause important changes in the results shown Figs. 5 and 6?

5. Cut \( DT \) in Half: Cut \( DT \) in half and repeat the simulation. If you see the same results, we know that 0.03125 months is sufficiently small for numerical accuracy.

6. Double \( DT \): Perhaps we can simulate the model with \( DT = 0.0625 \) instead? Select this value and simulate the model. Do you see the same results? Or perhaps you see ringing? (See page 227 in the book.) If so, which variable shows ringing? Why is this variable the first to exhibit ringing?

7. Sensitivity Test: \( \frac{r}{r} \) of decay in lower layer. This fraction is 0.7 in the base case simulation, but it could range from 0.5 to 0.9. What do you think will happen to the oxygen in the hypolimnion if this fraction were set to 0.9? Change the fraction and simulate the model. Are the results for oxygen in the hypolimnion significantly different than shown in Fig. 5? Are the new results realistic?

8. Sensitivity Test: month of fall turnover. What do you think will happen to the oxygen in the hypolimnion if the lake remains stratified for an additional month? Change the month of fall turnover from 9.0 to 10.0 and simulate the model. Are the results significantly different than shown in Fig. 5? Are the new results realistic?

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\(^8\) Check the footnote on page 5 of the previous BWeb document if you have forgotten how to assign a named variable to the initial value of the stock.
9. Model expansion for oxygen feedback on detritus decay: Exercises 7 and 8 will show negative values for the oxygen in the hypolimnion. This unrealistic result arises from the lack of feedback from the stock of oxygen to the consumption flow which drains the stock. (Recall exercise 3.9 from the book: there should always be feedback from a stock to the flows which drain the stock.) Expand the oxygen model by adding the variables shown in Fig. 9.

Figure 9. Extra variables to slow the detritus decay if there is insufficient oxygen.

Figure 10 shows the values of the decay multiplier as a function of the mg/L of oxygen in the lower layer. The multiplier is 1 when oxygen is 5 mg/L or greater. The value of 1 ensures that the normal decay rate applies.

The multiplier cuts the decay in half if the concentration is 2 mg/L. And if the concentration were to fall to zero, the multiplier also falls to zero, shutting down any decay of the detritus.

The indicated rate is the product of the Detritus C and the detritus decay rate, the same as the decay used previously. A binary variable is used to test the effect of the feedback. “O2 feedback to decay?” is 1 for yes, 0 for no. The binary variable makes it easier to run the model with and without the feedback effect. The equation for detritus C decay is:

\[
detritus \text{ C decay} = \text{O2 feedback to decay?} \times \text{indicated decay} \times \text{decay multiplier from O2 available} + (1-\text{O2 feedback to decay?}) \times \text{indicated decay}
\]
Simulate the new model with the “fr of decay in the lower layer” at 0.9. This is the value from exercise 7, so we should see the same results as exercise 7 if the binary variable is 0. Simulate the model with and without the feedback and create a comparison graph to verify the results in Figure 11.

![Compare Oxygen Concentrations (mg/L)](image)

**Figure 11.** Oxygen concentrations with and without the feedback effect on detritus decay.

The reduction in detritus decay at the end of the stratification periods will leave more detritus around for subsequent months when there is sufficient oxygen to fuel the decay. After several years, we could see higher detritus loads in the lake. With higher loads, we could see more detritus decay and more oxygen consumption. Oxygen concentrations would be reduced than there could be further reductions in the decay of the detritus. This sounds like a vicious circle that could cause much greater accumulation of detritus in the lake. Create a detritus comparison chart to verify the extra accumulation shown in Fig. 12.

![Compare Detritus Concentrations (umol/L)](image)

**Figure 12.** Detritus concentrations with and without the feedback effect on detritus decay.
10. Policy Test: Will oxygen concentrations improve if we eliminate the N inflow?

Simulate the model with the feedback effect on detritus decay and the “fr of decay in the lower layer” at 0.7. You should verify the results in Figure 13.

![Oxygen Concentration Graph](image)

Figure 13. Oxygen results when the feedback effect on detritus decay is included.

Notice that the oxygen does not fall below the threshold in first year. However, the second year shows a more serious decline, and there would be some fish mortality in the second year.

Now, suppose the fish mortality in the second year is recognized as a problem by the people in the watershed. And suppose the people are convinced that the nitrate and ammonium that is leaking into the lake are responsible for the problem. And finally, suppose the people take action to cut the N inflow completely to zero at the end of the second year.

Do you think the oxygen concentrations will return to the high levels needed to support the fish population? Perform the new simulation (with N inflow = 2 for the first half of the simulation, and N inflow = 0 for the second half). Does the simulation match your expectation? If not, explain which is more realistic - - your expectation or the model?

Sedimentation Exercises [under construction -- to be posted in Ver 5.1]

Figure 14 shows the stocks and flows to represent the accumulation of nitrogen in the sediments of the lake. This is the first step toward a “stand-alone model” that simulates nitrogen flows through the sediments. For testing purposes, we would set the variables in red with lookups to match results seen in previous simulations. The biomass N decay would remove detritus, but there is no need to follow it back to the nutrient pool. For example, the biomass N death could be based on results in the first year of the base case simulation of the model. Then, for a different test, the biomass N death could be based on the fourth year of the base case stimulation. The O2 in the Hypolimnion could be specified in a similar manner.
The Sedimentation of N moves nitrogen into the surface layer of the sediments. The nitrogen stored in the surface sediments are not in the nutrient pool. But they remain biologically available, and the N regeneration can return the nitrogen to the nutrient pool. The N regeneration corresponds to *internal loading*, and the threat of internal loading depends on the amount of surface sediment N.\(^9\)

The possibility of large accumulation of biologically available nitrogen in the surface sediments draws our attention to the regeneration rate. Regeneration is controlled by the oxygen concentration, as depicted in Figure 14. With high oxygen, there is little regeneration,\(^10\) and the N would remain in the surface sediments. Depending on the relative values of the burying rate and the liberation rate, the nitrogen could be transferred to the buried sediment. Under such conditions, the sediments would act as a nitrogen sink in the system. However, with anoxic conditions, the regeneration rate can be quite high, and the regeneration could send a surge of nitrogen into the nutrient pool. The sediments would act as a source of nitrogen.

The possibility of a sudden surge in *internal loading* will be the focus of the sedimentation exercises. It’s best to deal with sedimentation in two stages. The first stage is the development and testing of the stand alone model in Figure 14. Additional variables would be added to explain the rates, and the model would be tested in simulations with the lake moving from high oxygen to low oxygen conditions. The sedimentation sector could then be added to the eutrophication model, and the full model would be used to simulate the conditions that lead to a sudden surge in internal loading.

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\(^9\) Longmore (2006) gives an example from the Gippsland Lakes in Australia where the biologically available forms of ammonium are the equivalent of 4 years of ammonium inflow from the catchment.

\(^{10}\) The oxygen at the water-sediment interface inhibits the biogeochemical processes that transform nutrients into dissolvable forms (Beutel 1999).
Aeration Exercises [also under construction -- to be posted in Ver 5.1]

Aeration of the lake (by bubbling air into the water) is sometimes proposed to elevate the oxygen level. The aeration could protect the fish from the summer squeeze, and it could help the lake avoid the anoxic conditions that could trigger a surge of internal loading. Ver. 5.1 of these exercises will conclude with exercises to simulate the effect of different aeration strategies.

References

Beutel 1999

Hatch 2011
The Hatch Company has constructed a website for K-12 students and teachers. The website is known as the H2O University: [http://www.h2ou.com/index.htm](http://www.h2ou.com/index.htm)
Information on oxygen concentrations that are dangerous for fish are found on their page of Important Water Quality Factors: [http://www.h2ou.com/index.htm](http://www.h2ou.com/index.htm)

Longmore 2006

UNEP 2011
United Nations Environmental Program, Why is eutrophication such a serious pollution problem?

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Dissolved Oxygen: [http://www.waterontheweb.org/under/lakeecology/08_dissolvedoxygen.html](http://www.waterontheweb.org/under/lakeecology/08_dissolvedoxygen.html)
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Weiss 1970

Wetzel 2001